

## ANALYTIC CONTINUATION OF GENERALIZED TRIGONOMETRIC FUNCTIONS

PISHENG DING

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**ABSTRACT.** Via a unified geometric approach, certain generalized trigonometric functions with two parameters are analytically extended to maximal domains on which they are univalent. Some consequences are deduced concerning radius of convergence for the Maclaurin series, commutation with rotation, continuation beyond the domain of univalence, and periodicity.

### 1. INTRODUCTION

Inverses of functions of the form  $y \mapsto \int_0^y (1-x^q)^{-1/p} dx$  for  $y \in [0, 1]$  with  $p, q > 1$  have been of interest to analysts. See, for example, [1], [2], [5], [6]; also see [8] for an account of early work in this area. Herein, we study the complex-analytic aspects of a subclass of them and identify their maximal domain of univalence.

Throughout this note,  $n$  and  $k$  are integers with  $n > 2$  and  $1 \leq k < n$ .

For  $y \in [0, 1]$ , let

$$F_{n,k}(y) = \int_0^y \frac{1}{(1-x^n)^{k/n}} dx.$$

Denoting the number  $F_{n,k}(1)$  by  $\varphi_{n,k}$ , define  $S_{n,k} : [0, \varphi_{n,k}] \rightarrow [0, 1]$  to be the inverse of  $F_{n,k}$ . These functions  $S_{n,k}$  are often referred to as generalized sine functions. Primarily studied as real-valued functions, they are sometimes considered as analytic functions on  $F_{n,k}[\mathbb{D}]$  ( $\mathbb{D} = \{z \in \mathbb{C} \mid |z| < 1\}$ ) due to the fact that  $F_{n,k}$  is univalent as a complex-valued function on  $\mathbb{D}$  (by the Noshiro-Warschawski theorem in [4]).

In this article, we identify, for each  $n$  and  $k$ ,  $S_{n,k}$ 's "natural" domain of analyticity, which turns out to be a maximal domain on which  $S_{n,k}$  is univalent. Our unified treatment also encompasses the circular sine function (if we allow  $n = 2$ ), the historically important lemniscate sine function  $S_{4,2}$ , and the Dixon's elliptic function  $S_{3,2}$ . It is pertinent to note that, while we only treat the cases where  $k$  is an integer, the analysis herein is equally applicable if  $k$  is any positive real number less than  $n$ .

In §2, we introduce notation and state the main results, which are then proved in §3. In §4, we note several consequences, some of which concern further analytic continuation on larger domains beyond the domain of univalence.

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# Generalized Analytic Continuation

**Izrail' Moiseevich Gel'fand, Georgii  
Evgen'evich Shilov**



## **Generalized Analytic Continuation:**

Generalized Analytic Continuation William T. Ross, Harold S. Shapiro, 2002 The theory of generalized analytic continuation studies continuations of meromorphic functions in situations where traditional theory says there is a natural boundary This broader theory touches on a remarkable array of topics in classical analysis as described in the book The authors use the strong analogy with the summability of divergent series to motivate the subject They are careful to cover the various types of continuations attempting to unify them and suggesting some open questions The book also addresses the role of such continuations in approximation theory and operator theory The introductory overview provides a useful look at the history and context of the theory

**General Principles of Quantum Field Theory** N.N. Bogolubov, Anatoly A. Logunov, A.I. Oksak, I. Todorov, 2012-12-06 The majority of the memorable results of relativistic quantum theory were obtained within the framework of the local quantum field approach The explanation of the basic principles of the local theory and its mathematical structure has left its mark on all modern activity in this area Originally the axiomatic approach arose from attempts to give a mathematical meaning to the quantum field theory of strong interactions of Yukawa type The fields in such a theory are realized by operators in Hilbert space with a positive Poincare invariant scalar product This classical part of the axiomatic approach attained its modern form as far back as the sixties It has retained its importance even to this day in spite of the fact that nowadays the main prospects for the description of the electro weak and strong interactions are in connection with the theory of gauge fields In fact from the point of view of the quark model the theory of strong interactions of Wightman type was obtained by restricting attention to just the physical local operators such as hadronic fields consisting of fundamental quark fields acting in a Hilbert space of physical states In principle there are enough such physical fields for a description of hadronic physics although this means that one must reject the traditional local Lagrangian formalism The connection is restored in the approximation of low energy phenomenological Lagrangians

**Generalized Analytic Functions. English Translation Editor: Ian N. Sneddon** Il'ia Nestorovich Vekua, 1962 **Generalized Analytic Functions with Applications to Singular Ordinary and Partial Differential Equations** Bogdan Ziemian, 1996

**Symposia on Theoretical Physics and Mathematics 8** Alladi Ramakrishnan, 2013-03-09 This volume comprises the lectures given at the Fifth Anniversary Symposium held at the Institute of Mathematical Sciences Madras India during January 1967 Professor Dallaporta of Padua delivered the inaugural address on the fundamental problem of quasars whose study appears to hold implications for cosmology He presented a critical review of several attempts to understand their exceptionally large red shifts and also discussed the physical theories concerning the cause of the explosions which give rise to the quasars and to the tremendous energy output they require questions which still remain unanswered He stated in concluding that we may have to invoke certain aspects of the present theories of elementary particles in order to unravel these mysteries Professor Mercier well known for his studies on the philosophical foundations of modern physics critically

examined the various attempts such as that of Einstein to formulate a unified field theory Analytic Continuation and q-Convexity Takeo Ohsawa, Thomas Pawlaschyk, 2022-06-02 The focus of this book is on the further development of the classical achievements in analysis of several complex variables the analytic continuation and the analytic structure of sets to settings in which the  $q$  pseudoconvexity in the sense of Rothstein and the  $q$  convexity in the sense of Grauert play a crucial role After giving a brief survey of notions of generalized convexity and their most important results the authors present recent statements on analytic continuation related to them Rothstein 1955 first introduced  $q$  pseudoconvexity using generalized Hartogs figures S odkowski 1986 defined  $q$  pseudoconvex sets by means of the existence of exhaustion functions which are  $q$  plurisubharmonic in the sense of Hunt and Murray 1978 Examples of  $q$  pseudoconvex sets appear as complements of analytic sets Here the relation of the analytic structure of graphs of continuous surfaces whose complements are  $q$  pseudoconvex is investigated As an outcome the authors generalize results by Hartogs 1909 Shcherbina 1993 and Chirka 2001 on the existence of foliations of pseudoconcave continuous real hypersurfaces by smooth complex ones A similar generalization is obtained by a completely different approach using  $L^2$  methods in the setting of  $q$  convex spaces The notion of  $q$  convexity was developed by Rothstein 1955 and Grauert 1959 and extended to  $q$  convex spaces by Andreotti and Grauert 1962 Andreotti Grauert's finiteness theorem was applied by Andreotti and Norguet 1966 1971 to extend Grauert's solution of the Levi problem to  $q$  convex spaces A consequence is that the sets of  $q-1$  cycles of  $q$  convex domains with smooth boundaries in projective algebraic manifolds which are equipped with complex structures as open subsets of Chow varieties are in fact holomorphically convex Complements of analytic curves are studied and the relation of  $q$  convexity and cycle spaces is explained Finally results for  $q$  convex domains in projective spaces are shown and the  $q$  convexity in analytic families is investigated **Lectures Presented at the Anniversary Symposium** Institute of Mathematical Sciences (India), 1968

Complex Variables Robert B. Ash, 2014-05-10 Complex Variables deals with complex variables and covers topics ranging from Cauchy's theorem to entire functions families of analytic functions and the prime number theorem Major applications of the basic principles such as residue theory the Poisson integral and analytic continuation are given Comprised of seven chapters this book begins with an introduction to the basic definitions and concepts in complex variables such as the extended plane analytic and elementary functions and Cauchy Riemann equations The first chapter defines the integral of a complex function on a path in the complex plane and develops the machinery to prove an elementary version of Cauchy's theorem Some applications including the basic properties of power series are then presented Subsequent chapters focus on the general Cauchy theorem and its applications entire functions families of analytic functions and the prime number theorem The geometric intuition underlying the concept of winding number is emphasized The linear space viewpoint is also discussed along with analytic number theory residue theory and the Poisson integral This book is intended primarily for students who are just beginning their professional training in mathematics Encyclopaedia of Mathematics Michiel

Hazewinkel,2013-12-20    **Mathematics of the USSR. ,1976    Lectures ,1970    Generalized Functions: Properties and operations, by I. M. Gel'fand and G. E. Shilov, translated by E. Saletan** Izrail' Moiseevich Gel'fand,Georgii Evgen'evich Shilov,1964    Dynamical Systems George David Birkhoff,1927    Ananda Rau Memorial Volume V. K. Balachandran,C. T. Rajagopal,M. S. Rangachari,1969    **Colloquium Publications** ,1927    *Symposia on Theoretical Physics and Mathematics* ,1970    Theory and Application of the Boltzmann Equation Carlo Cercignani,1975 Appendix after each chapter    Acta Scientiarum Mathematicarum József Attila Tudományegyetem,1973    **Fourier Analysis and Approximation Theory** György Alexits,Paul Turán,1978    **Pacific Journal of Mathematics** ,1972

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